

# Data Science Toolbox Question Sheet

## 03.1 Latent Spaces and PCA

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### Block 3

#### Short questions

1. If we use the identity that  $C = Cov(X) = (1/(n - 1))XX^T$ , what assumptions have we made about  $X$ ?
2. We project the data  $X$  onto the subspace  $U$  using the projection  $P$  using  $XP$ . What can you say about  $(XP)P$ ?
3. In what sense is  $Cov(X) = U\Sigma U^T$  truncated to low rank  $K$  the “best” low rank summary of  $X$ ?
4. Give a high-level explanation for why minimising the mean squared error and maximising the variance of a low-dimensional representation of a matrix  $X$  into  $U$ ,  $D$  and  $V$  leads to the same representation, the SVD.
5. What is an eigenvalue? What is an eigenvector?
6. What is the variance explained by the  $k$ -th eigenvector?
7. What is the relationship between singular values and eigenvalues?